11-5
Infinite Geometric Series

Challenge:

Zeno's Paradox

$8 + 4 + 2 + 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots$

$S_1 = 8$
$S_2 = 12$
$S_3 = 14$
$S_4 = 15$
$S_5 = 15.5$
$S_6 = 15.75$
$S_7 = 15\frac{7}{8}$
$S_8 = 15\frac{15}{16}$
$S_9 =$
$S_{10} =$

$S_\infty = \frac{8(1)}{1 - \frac{1}{2}}$

$S = 16$

$$S_\infty = \frac{a_1}{1-r} \qquad |r| < 1$$

- Convergent--has a sum $|r| < 1$

Divergent--no sum $|r| \geq 1$

ex 1:

$24 + 12 + 6 + \ldots$

$r = \frac{1}{2}$ $S_\infty = \frac{24}{1-\frac{1}{2}} = 48$

ex 2:

$3 + 1 + \frac{1}{3} + \frac{1}{9} + \ldots$

$r = \frac{1}{3}$

$S_\infty = \frac{3}{1-\frac{1}{3}} = 4.5$

ex 3:

$2 - 2 + 2 - 2 + \ldots$

$r = -1$

divergent No sum

ex 4:

$1 + 2 + 4 + 8 + \ldots$

$r = 2$

div.

No Sum

Do:

1. $\frac{3}{10}+\frac{3}{100}+\frac{3}{1000}+....$

$r=\frac{1}{10}$

2. $24 + 9.6+ 3.84 + ...$

$r = .4$ $S_\infty = \frac{24}{1-.4} = 40$

Repeating Decimals

1. Express as an infinite series

2. Convert to a fraction

ex

$.\overline{5}$

$r = .1$ $S_\infty = \frac{.5}{1-.1} = \frac{.5}{.9} = \frac{5}{9}$

Repeating Decimal

ex

$.\overline{900}$

$r = .001$ $S_\infty = \frac{.9}{1-.001} = \frac{100}{111}$

Do:

1. $.\overline{45}$ $r = .01$ $\frac{.45}{1-.01} = \frac{5}{11}$

2. $.\overline{9} = 1$

$r = .1$ $S_\infty = \frac{.9}{1-.1} = \frac{.9}{.9} = 1$

$.\overline{3} + .\overline{3} + .\overline{3} = .\overline{9}$

$\frac{1}{3} + \frac{1}{3} + \frac{1}{3} = \frac{3}{3} = 1.$

Solve for x if $S_\infty = 3$.

$x + x^2 + x^3 + \ldots$

$r = \frac{x^2}{x} = x \qquad \frac{3}{1} = \frac{x}{1-x}$

$3(1-x) = x$

$3 - 3x = x$

$3 = 4x$

$\frac{3}{4} = x$

Sigma Notation

$\sum_{n=1}^{\infty} 5\left(\frac{1}{2}\right)^{n-1}$ $a_1 r^{n-1}$

$a_1 = 5 \qquad S_\infty = \frac{5}{1-\frac{1}{2}} = 10$

$r = \frac{1}{2}$

p602

13. **CLOCKS** Jasmine's old grandfather clock is broken. When she tries to set the pendulum in motion by holding it against the side of the clock and letting it go, it first swings 24 centimeters to the other side, then 18 centimeters back, then 13.5 centimeters, and so on. What is the total distance that the pendulum swings?

$a_1 = 24 \qquad r = \frac{18}{24} = \frac{3}{4}$

$a_2 = 18$

$a_3 = 13.5 \qquad S_\infty = \frac{24}{1-\frac{3}{4}} = 96 \text{ cm}$

35. **AVIATION** A hot-air balloon rises 90 feet in its first minute of flight. In each succeeding minute, it rises only 90% as far as it did during the preceding minute. What is the final height of the balloon?

$a_1 = 90$

$r = .9$ $S_\infty = \frac{90}{1 - .9}$

900 ft

Hw
p602-603
19-31odd, 32, 36, 37, 40,44